$\boxed{5}$ $P = 1$

$E_0 = E_0 = |P - P_0| = 0.2$

$E_1 = |P - P_1| = 0.006060606$

$E_2 = |P - P_2| = 0.000006087$

$E_3 = |P - P_3| = 0$

$$\cancel{E_{\alpha}} \; E_1 = \lim_{n \to \infty} \frac{|P - P_{n+1}|}{|P - P_n|^{R}} = A$$

11. $\frac{7}{17} + \frac{81}{13} + \frac{801}{19}$

$(0.4118 + 6.231) + 42.16$

$6.643 + 42.16 = \boxed{48.80}$

12. $-2 + 2 + 2a + 1 = 1$

$\Rightarrow \boxed{a = 0}$

$$S'(x) = \begin{cases} -6x^2 + 4x & , 0 \le x \le 1 \\ 21(x-1)^2 - 8(x-1) + b & , 1 < x \le 2 \end{cases}$$

$S'_0(x_1) = S'_1(x_1)$

$S'(1) = -6 + 4 = b \Rightarrow \boxed{b = -2}$

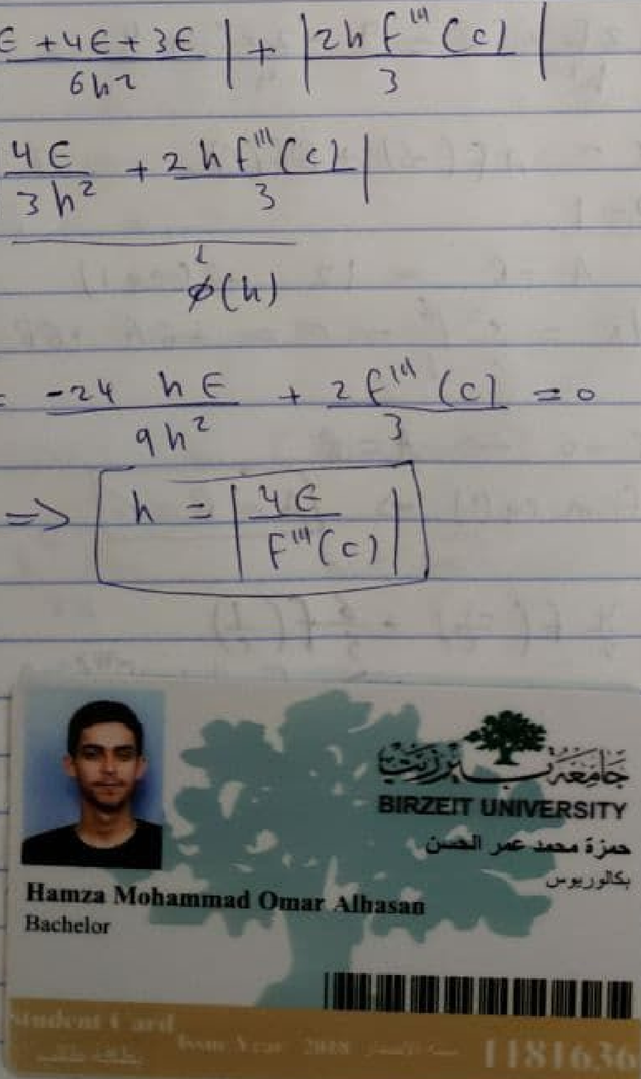
13. $f''(x_0) = \frac{f_3 - 4f_0 + 3f_{-1}}{6h^2} - \frac{2hf'''(c)}{3}$

$|E_{tot}| \le \left|\frac{\epsilon + 4\epsilon + 3\epsilon}{6h^2}\right| + \left|\frac{2hf'''(c)}{3}\right|$

$\le \underbrace{\left|\frac{4\epsilon}{3h^2} + \frac{2hf'''(c)}{3}\right|}_{\phi(h)}$

$\phi'(h) = \frac{-24h\epsilon}{9h^2} + \frac{2f'''(c)}{3} = 0$

$\Rightarrow \boxed{h = \left|\frac{4\epsilon}{f'''(c)}\right|}$



$\boxed{14}$ $\int_{-1}^{1} x^2 e^{x^2} dx$, Simpsen's Rule

$-1 \quad 0 \quad 1$

$h = 1$

$= \frac{h}{3}(f_0 + 4f_1 + f_2)$

$f_0 = e = f(-1)$

$f_1 = f(0) = 0$

$f_2 = f(1) = e$

$\Rightarrow \int_{-1}^{1} x^2 e^{x^2} dx = \frac{1}{3}(e + e)$

$= \boxed{1.81219}$

$\boxed{15}$ CD of order $O(h^2)$:

$f'(4)$ and $f''(4)$

$(0,1)$

$(2,4)$

$(4,6)$

$(6,9)$

$\Rightarrow \underline{h = 2}$

$f'(4) = \frac{f_1 - f_{-1}}{2h} = \frac{f(4+2) - f(4-2)}{4}$

$= \frac{9 - 4}{4} = \boxed{\frac{5}{4}}$

$f''(4) = \frac{f_1 - 2f_0 + f_{-1}}{h^2} = \frac{9 - 2\times 6 + 4}{4} = \boxed{\frac{1}{4}}$

$\boxed{16}$ $\int_{-6}^{6} f(x)dx \approx Af(-6) + Bf(6)$

$\underline{DP = 1}$

$\int_{-6}^{6} dx = A + B = 12$ — (eq 1)

$\int_{-6}^{6} x\, dx = \frac{x^2}{2}\Big|_{-6}^{6} = 0 = -6A + 6B$ — (eq 2)

$-6A + 6B = 0 \Rightarrow A = B$

from eq(1) $\Rightarrow \boxed{A = B = 6}$

$\boxed{17}$ $\int_{-1}^{1} F(x)dx \approx \frac{4}{5} f\left(\frac{-1}{2}\right) + \frac{6}{5} f\left(\frac{1}{3}\right)$

$\underline{DP=1}$ $\Rightarrow E = K f''(c)$

let $f(x) = (x+1)^2 \Rightarrow f'(x) = 2(x+1)$

$f''(c) = 2$

$\Rightarrow E = 2K$

$E =$ True - Estimite $= \int_{-1}^{1}(x+1)^2 dx - \frac{4}{5}\left(\frac{1}{4}\right) - \frac{6}{5}\left(\frac{16}{9}\right) = 2K$

$\downarrow$

$= \frac{8}{3}$

$\Rightarrow 2K = \frac{1}{3} \Rightarrow K = \frac{1}{6}$

$\Rightarrow \boxed{E = \frac{1}{3}}$ $\Rightarrow \boxed{E = \frac{1}{6} f''(c)}$

$\boxed{9}$ $x_0 = 3, \; x_1 = 4, \; x_2 = 6, \; x_3 = 8$

$$L_{3,2}(x) = \frac{\prod_{j=0, j\neq 2}^{3} (x - x_j)}{\prod_{j=0, j\neq 2}^{3} (x_k - x_j)}$$

$$= \frac{1}{12}(8-x)(x-4)(x-3)$$

$$\boxed{L_{3,2}(5) = \frac{1}{2}}$$

$\boxed{10}$ $P_2(x) = y_0 L_{2,0} + y_1 L_{2,1} + y_2 L_{2,2}$

$$L_{2,0} = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \Rightarrow \begin{array}{l} - : 4 \\ x : 2 \\ \div : 1 \end{array}$$

$\Rightarrow \text{Cost}(L_{2,0}) = 7$

Clearly

and Similarcily, cost of $L_{2,1}, L_{2,2} = 7$

$\Rightarrow$ Cost of the L's $= 7 \times 3 = 21$

cost of P_2: $x : 3$

$+ : 2$

$\Rightarrow$ Total cost $= 3 + 2 + 21 = \boxed{26}$

$\boxed{11}$ $f(x) = \ln(x+1) \rightarrow [0.1, 0.4]$, uniform

$$|E_3(x)| \le \frac{h^4 M_4}{24}, \quad h = \frac{0.4 - 0.1}{3} = 0.1$$

$M_4 = \text{Max}\, |f^{(4)}(x)|$

$$f'(x) = \frac{1}{x+1}, \quad f''(x) = \frac{-1}{(1+x)^2}, \quad f'''(x) = \frac{2}{(1+x)^3}$$

$$|f^{(4)}(x)| = \frac{6}{(1+x)^4} \text{ is max at } x = 0.1$$

$$M_4 = \frac{6}{(1.1)^4} = 4.0981$$

$$|E_3(x)| \le \frac{(0.1)^4 (4.0981)}{24} = 1.70754 \times 10^{-5}$$

$\boxed{7}$ $y = x^3$ closest to point $(1, 2)$

$D^2 = (x-1)^2 + (x^3 - 2)^2$

$2DD' = 2(x-1) + 6x^2(x^3-2)$

$=$

let $f(x) = (x-1) + 3x^2(x^3-2)$

we need $f(x) = 0$ to find the closest point

$f(x) = 3x^5 - 6x^2 + x - 1$

$f(0) = -1$, $\quad f(2) = 73$

$f(x)$ is cont. and diff on $[0, 2]$

$\Rightarrow$ Bolzano $\Rightarrow \exists c \in [0, 2]$ s.t $f(c) = 0$

$c_0 = 1 \Rightarrow f(c_0) = f(1) = -3 < 0$

$c_1 = 1.5 \Rightarrow f(1.5) = 9.78 > 0$

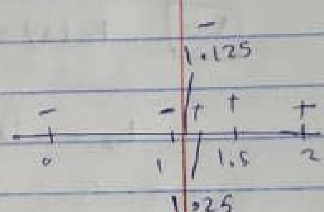
$c_2 = 1.25 \Rightarrow f(1.25) = 0.03.. > 0$

$c_3 = \frac{1.25+1}{2} = 1.125 \Rightarrow f(c_3) = -2.06 < 0$ / $c_4 = \frac{1.125 + 1.25}{2}$

$\boxed{c_4 = 1.1875}$ ✓

the point is :

$(1.1875, 1.67456)$



There is no need to specify the # of digits used. ☺

I got better answer ☺

However, if you want 2 digits :

$\Rightarrow \frac{1.2+1}{2} = \boxed{1.1}$ ✓

$\Rightarrow$ Point is $(1.1, 1.331)$

$\boxed{8}$ $f[1.3, 2.4, 3.6]$, $\quad f(x) = x^2$

~~$f(x_0) = y_0 = a_0$~~

~~$f(x_1)$ $f[x_0, x_1] = a_1$~~

let $A = \begin{bmatrix} f(x_0) = y_0 = a_0 & 0 & 0 \\ f(x_1) = y_1 & f[x_0, x_1] = a_1 & 0 \\ f(x_2) = y_2 & f[x_1, x_2] & f[x_0, x_1, x_2] = a_2 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} 1.69 & 0 & 0 \\ 5.76 & 3.7 & 0 \\ 12.96 & 6 & 1 \end{bmatrix}$

$f[x_0, x_1] = a_1 = \frac{y_1 - y_0}{x_1 - x_0}$, $\quad f[x_1, x_2] = \frac{y_2 - y_1}{x_2 - x_1}$

$f[x_0, x_1, x_2] = a_2 = \left(\frac{y_1 - y_0}{x_1 - x_0} + \frac{y_2 - y_1}{x_2 - x_1}\right) \div (x_2 - x_0)$

$\Rightarrow f[1.3, 2.4, 3.6] = a_2 = 1$

$\boxed{2}$ $[4,6]$, $\delta = 10^{-5}$; Bisection Method

$$n > \frac{\ln\left(\frac{b-a}{\delta}\right)}{\ln 2} - 1$$

$$> \frac{\ln(2/10^{-5})}{\ln 2} - 1$$

$$n > 16.61 \Rightarrow \boxed{n = 17}$$

$\boxed{3}$ Secant , $p_0 = 1$, $p_1 = 1.5$

eq : $f(x) = x^5 - x - 4$

$\cancel{f'(x) = 5x^4 - 1}$

$$p_2 = p_1 - \left(\frac{p_1 - p_0}{f(p_1) - f(p_0)}\right) f(p_1)$$

$$= 1.5 - \left(\frac{0.5}{2.09375 + 4}\right) \times 2.09375$$

$$\boxed{p_2 = 1.328205}$$

$\boxed{4}$ $x = \frac{10}{x} + 3 \Rightarrow x^2 - 3x - 10 = 0$ $\rightarrow p_1 = -2$

$\cancel{x = \frac{+3 \pm \sqrt{4+40}}{2}}$ $p_2 = 5$

$$|g'(x)| = \left|\frac{-10}{x^2}\right| = \frac{10}{x^2}$$

$g'(-2) = \frac{5}{2} > 1$ $\cancel{x = 1 - \sqrt{11} = -2.31662 = p_1}$

$g'(5) = \frac{2}{5} < 1$ $\cancel{x = 1 + \sqrt{11} = 4.31662 = p_2}$

$g'(p_1) = g'(\cancel{-2.31662}\,{-2}) = \cancel{1.863325}$ $\Rightarrow$ FPI div and p_1 is repellel

$g'(p_2)$ $= \frac{2}{5}$ $\cancel{0.5326675} < 1 \rightarrow$ FPI conv. and p_2 is attracter

$\boxed{p_1 \text{ is repellel} \Rightarrow p_1 = -2}$

$\boxed{6}$ $f(x) = (x+3)^3(x-1)$

clearly $p = -3$, $\underline{p = 1}$

$f'(x) = 3(x-1)(3+x)^2 + (3+x)^3$

$f'(1) = 64 \neq 0 \Rightarrow M = 1$ (simple root)

$\Rightarrow R = 2 \Rightarrow A = \left|\frac{f''(p)}{2f'(p)}\right|$

$f''(x) = 6(x-1)(x+3) + 6(x+3)^2$

$f''(1) = 96 \Rightarrow A = \left(\frac{96}{2 \times 64}\right) = \boxed{\frac{3}{4} = A}$